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Interval arithmetic in calculations

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Abstract: Interval arithmetic is the mathematical structure, which for real intervals defines operations analogous to ordinary arithmetic ones. This field of mathematics is also called interval analysis or interval calculations. The given math model is convenient for investigating various applied objects: the quantities, the approximate values of which are known; the quantities obtained during calculations, the values of which are not exact because of rounding errors; random quantities. As a whole, the idea of interval calculations is the use of intervals as basic data objects. In this paper, we considered the definition of interval mathematics, investigated its properties, proved a theorem, and showed the efficiency of the new interval arithmetic. Besides, we briefly reviewed the works devoted to interval analysis and observed basic tendencies of development of integral analysis and interval calculations.

Keywords: Interval methods; interval numbers; sub-distributivity; interval-valued function

1 Introduction

Along with many achievements in the sphere of science and technology, in the middle years of the XX century there began introduction of wider paradigm of realizing the uncertainty nature, which had been developed in various spheres of applied mathematics before. It was based on the interpretation of uncertainty in the wider context broadening the concept of randomness: non-uniqueness of possible outcomes, semantic variability, multicriteriality for optimization problems. New approaches to describing uncertainty caused the appearance of concepts of multivalued logics and sub-definite models, the theory of fuzzy sets

and numbers, and besides interval analysis, the subject of which is solving problems with interval uncertainties and ambiguity in data arising both when defining a problem and at intermediate stages of calculations [3, 4]. Engineers and constructors pay close attention to the interval representation of uncertainty factors as less restrictive and more adequate description of initial conditions when defining engineer problems practically. Such uncertainty is called an interval uncertainty since it specifies only limits of possible values of some quantity (or its ranging limits), the information of which is incomplete. The interval uncertainty of a quantity expressed by its boundary values can be considered as an interval parameter, which has the following features: any quantity with an interval uncertainty can be represented only by its boundary values, *i.e.* limits of possible values (or ranging limits) of this quantity; the width of the interval, by which such quantity is expressed, is a natural measure of its uncertainty (ambiguity); the result of arithmetic operations over quantities with an interval uncertainty is also an interval uncertainty.

Interval analysis is actively developing in many countries currently. Interval methods have been outside the scope of purely theoretical research for a long time, and now they are rather widely used in practice by means of the appropriate software. It has resulted in appearance of interval arithmetic, interval algebra, interval topology, interval methods of solving problems of computational mathematics, optimal control, stability problems, etc.

The goal of this paper is development of the new interval arithmetic including nonuniform distribution of values within an interval.

We gave the definition of a new interval mathematics, investigated its properties. Based on solving some classical problems we showed the efficiency of the interval mathematics introduced.

2 Materials and Methods

Interval methods originated as a means of machine rounding error check on a computer, and then changed into one of chapters of modern applied mathematics.


The first publication devoted to interval analysis was the work by R.E.Mur, published in 1966 [1, 12].

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U.I.Shokin [2] in 1981 used to expound fundamentals and methods of integral analysis. Then, in 1982 T.I.Nazarenko and L.V.Marchenko [3] published a training aid on interval methods, and in 1986 the survey monograph by S.A.Kalmykov, U.I.Shokin, Z.Kh.Yuldashev [4] was published. In these works the authors systematically expounded fundamentals and methods of interval analysis, thoroughly introduced interval arithmetic, and together with “classical interval arithmetic” considered a number of its modifications and generalizations. Besides, the works by S.P.Sharoy, Hayes B., van Emden [6–8] have been devoted to interval arithmetic.

The ideas of interval analysis are of practical interest, but one can face great difficulties when realizing them. The study revealed that one could not just use traditional methods when manipulating interval numbers. Due to this there appeared many publications, in which we reconsidered various numerical methods for interval data. Such reconsideration can be found in the works by B.S.Dobronetz and V.V.Shaidurov [5].

However, when applying interval mathematics researchers experience difficulties when solving cumbersome interval equations, besides, the obtained solutions are “over-sufficient”, which is a severe restriction.

Let us introduce the formal concept of an interval a in the following form:

$$a = [\bar{a} - \varepsilon_a, \bar{a} + \varepsilon_a] = (\bar{a}, \varepsilon_a); \tag{1}$$

where \bar{a} is the center of the interval (or the mean), ε_a is the width of the interval (or dispersion). Let us denote the set of all such intervals as $I_{Bep}(R)$.

Let a, b, c be intervals from $I_{Bep}(R)$. Let us introduce the following interval arithmetic operations (assumed that intervals are independent normally distributed quantities):

1. Addition of two intervals $a, b \in I_{Bep}(R)$: $c = a + b$,

$$\bar{c} = \bar{a} + \bar{b}; \varepsilon_c = \sqrt{\varepsilon_a^2 + \varepsilon_b^2}; \tag{2}$$

2. Subtraction of two intervals $a, b \in I_{Bep}(R)$: $c = a - b$,

$$\bar{c} = \bar{a} - \bar{b}; \varepsilon_c = \sqrt{\varepsilon_a^2 + \varepsilon_b^2}; \tag{3}$$

3. Multiplication of two intervals $a, b \in I_{Bep}(R)$: $c = a \cdot b$,

$$\bar{c} = \bar{a} \cdot \bar{b}; \varepsilon_c = \sqrt{a^{-2} \cdot \varepsilon_b^2 + b^{-2} \cdot \varepsilon_a^2}; \tag{4}$$

4. Inverse interval $a \in I_{Bep}(R)$: $c = \frac{1}{a}$;

$$\bar{c} = \frac{1}{\bar{a}}; \varepsilon_c = \frac{\varepsilon_a}{\bar{a}^2} \tag{5}$$

5. Division of two intervals $a, b \in I_{Bep}(R)$: $c = \frac{1}{b}$;

$$\bar{c} = \frac{\bar{a}}{\bar{b}}; \varepsilon_c = \sqrt{\frac{\bar{a}^2 \cdot \varepsilon_b^2}{\bar{b}^4} + \frac{\varepsilon_a^2}{\bar{b}^2}} \tag{6}$$

3 Theory and Calculations

Theorem. Let a, b, c be intervals from $I_{Bep}(R)$. Then the following hold

- 1) $a + b = b + a$; $a \cdot b = b \cdot a$; (commutativity); (7)
- 2) $(a + b) + c = a + (b + c)$; (8)
 $(a \cdot b) \cdot c = a \cdot (b \cdot c)$; (associativity);
- 3) $x = [0, 0]$ and $y = [1, 1]$ the only neutral elements of addition and multiplication respectively, that is $a = x + a = a + x$ for all intervals a ; $a = y \cdot a = a \cdot y$ for all intervals a ;
- 4) an arbitrary interval a , which is not point, has an inverse neither with respect to addition nor with respect to multiplication. Nevertheless,

$$0 \in a - a; 1 \in \frac{a}{a}.$$

- 5) provided $\bar{b} \cdot \bar{c} \leq 0$, sub-distributivity holds

$$a \cdot (b + c) \subseteq a \cdot b + a \cdot c. \tag{9}$$

Proof:

- 1) Let us prove commutativity. Let $c = a + b$; $d = b + a$. Then formula (2) implies $\bar{c} = \bar{a} + \bar{b}$; $\varepsilon_c = \sqrt{\varepsilon_a^2 + \varepsilon_b^2}$; $\bar{d} = \bar{b} + \bar{a}$; $\varepsilon_d = \sqrt{\varepsilon_b^2 + \varepsilon_a^2}$; therefore, $c = d$. Let $c = a \cdot b$; $d = b \cdot a$. From formula (2) and (4) it follows that $\bar{c} = \bar{a} \cdot \bar{b}$; $\varepsilon_c = \sqrt{\bar{a}^2 \cdot \varepsilon_b^2 + \bar{b}^2 \cdot \varepsilon_a^2}$; $\bar{d} = \bar{b} \cdot \bar{a}$; $\varepsilon_d = \sqrt{\bar{b}^2 \cdot \varepsilon_a^2 + \bar{a}^2 \cdot \varepsilon_b^2}$; therefore, $c = d$.
- 2) Let us prove associativity for addition. Let $d = a + b$; $e = b + c$; $f = d + c$; $g = a + e$. Then substituting these into formula (2) we obtain: $\bar{d} = \bar{a} + \bar{b}$; $\varepsilon_d = \sqrt{\varepsilon_a^2 + \varepsilon_b^2}$; $\bar{e} = \bar{b} + \bar{c}$; $\varepsilon_e = \sqrt{\varepsilon_b^2 + \varepsilon_c^2}$; $\bar{f} = \bar{d} + \bar{c} = \bar{a} + \bar{b} + \bar{c}$; $\varepsilon_f = \sqrt{\varepsilon_d^2 + \varepsilon_c^2} = \sqrt{\varepsilon_a^2 + \varepsilon_b^2 + \varepsilon_c^2}$; $\bar{g} = \bar{a} + \bar{e} = \bar{a} + \bar{b} + \bar{c}$; $\varepsilon_g = \sqrt{\varepsilon_a^2 + \varepsilon_e^2} = \sqrt{\varepsilon_a^2 + \varepsilon_b^2 + \varepsilon_c^2}$; therefore, $f = g$.

Let us prove associativity for multiplication. Let $d = a \cdot b$; $e = b \cdot c$; $f = d \cdot c$; $g = a \cdot e$. Then substituting these into formula (4) we have:

$$\begin{aligned} \bar{d} &= \bar{a} \cdot \bar{b}; \\ \varepsilon_d &= \sqrt{\bar{a}^2 \cdot \varepsilon_b^2 + \bar{b}^2 \cdot \varepsilon_a^2}; \quad \bar{e} = \bar{b} \cdot \bar{c}; \\ \varepsilon_e &= \sqrt{\bar{b}^2 \cdot \varepsilon_c^2 + \bar{c}^2 \cdot \varepsilon_b^2}; \quad \bar{f} = \bar{d} \cdot \bar{c} = \bar{a} \cdot \bar{b} \cdot \bar{c}; \end{aligned}$$

$$\begin{aligned} \varepsilon_f &= \sqrt{\bar{d}^2 \cdot \varepsilon_c^2 + \bar{c}^2 \cdot \varepsilon_a^2} \\ &= \sqrt{\bar{a}^2 \cdot \bar{b}^2 \cdot \varepsilon_c^2 + \bar{c}^2 \cdot \bar{a}^2 \cdot \varepsilon_b^2 + \bar{c}^2 \cdot \bar{b}^2 \cdot \varepsilon_a^2}; \\ \bar{g} &= \bar{a} \cdot \bar{e} = \bar{a} \cdot \bar{b} \cdot \bar{c}; \\ \varepsilon_g &= \sqrt{\bar{a}^2 \cdot \varepsilon_e^2 + \bar{e}^2 \cdot \varepsilon_a^2} \\ &= \sqrt{\bar{a}^2 \cdot \bar{b}^2 \cdot \varepsilon_c^2 + \bar{a}^2 \cdot \bar{c}^2 \cdot \varepsilon_b^2 + \bar{b}^2 \cdot \bar{c}^2 \cdot \varepsilon_a^2}; \end{aligned}$$

therefore, $f = g$.

- 3) Let $c = x+a; d = a+x$, where $x = [0, 0] = (0, 0)$. Then $\bar{c} = \bar{x} + \bar{a} = 0 + \bar{a} = \bar{a}; \varepsilon_c = \sqrt{\varepsilon_x^2 + \varepsilon_a^2} = \sqrt{0 + \varepsilon_a^2} = \varepsilon_a; \bar{d} = \bar{a} + \bar{x} = \bar{a} + 0 = \bar{a}; \varepsilon_d = \sqrt{\varepsilon_a^2 + \varepsilon_x^2} = \sqrt{\varepsilon_a^2 + 0} = \varepsilon_a$; therefore, $c = d = a$.

Let $c = y \cdot a; d = a \cdot y$, where $y = [1, 1] = (1, 0)$. Then

$$\begin{aligned} \bar{c} &= \bar{y} \cdot \bar{a} = 1 \cdot \bar{a} = \bar{a}; \\ \varepsilon_c &= \sqrt{\bar{y}^2 \cdot \varepsilon_a^2 + \bar{a}^2 \cdot \varepsilon_y^2} = \sqrt{1 \cdot \varepsilon_a^2 + \bar{a}^2 \cdot 0} = \varepsilon_a; \\ \bar{d} &= \bar{a} \cdot \bar{y} = \bar{a} \cdot 1 = \bar{a}; \\ \varepsilon_d &= \sqrt{\bar{a}^2 \cdot \varepsilon_y^2 + \bar{y}^2 \cdot \varepsilon_a^2} = \sqrt{\bar{a}^2 \cdot 0 + 1 \cdot \varepsilon_a^2} = \varepsilon_a; \end{aligned}$$

therefore, $c = d = a$.

- 4) Let a be an arbitrary non-point interval, that is $\varepsilon_a \neq 0$; b is inverse to a interval with respect to addition. Then the interval $c = a - b$ is to be a zero point interval $c = (0, 0)$. However, according to the definition $\bar{c} = \bar{a} - \bar{b}; \varepsilon_c = \sqrt{\varepsilon_a^2 + \varepsilon_b^2}$; this is possible only if $\bar{a} = \bar{b}$ and $\varepsilon_a = 0, \varepsilon_b = 0$, which contradicts the hypothesis that the interval a is not a point interval.

Let $c = a - a$, then $\bar{c} = 0$ and $\varepsilon_c = \varepsilon_a \cdot \sqrt{2}$, that is the interval c contains the point 0. Let a be an arbitrary non-point interval, i.e. $\varepsilon_a \neq 0$; b is inverse to a interval with respect to multiplication, and $\bar{b} \neq 0$. Then the interval $c = \frac{a}{b}$ is to be a unit point interval $c = (1, 0)$. However, according to the definition $\bar{c} = \frac{\bar{a}}{\bar{b}}; \varepsilon_c = \sqrt{\frac{\varepsilon_a^2}{\bar{b}^2} + \frac{\varepsilon_b^2}{\bar{b}^2}}$ this is possible only if $\bar{a} = \bar{b}$ and $\varepsilon_a = 0, \varepsilon_b = 0$, which contradicts the hypothesis that the interval a is not a point interval.

Let $c = \frac{a}{a}$ then $\bar{c} = 1$ and $\varepsilon_c = \varepsilon_a \cdot \sqrt{\frac{2}{\bar{a}^2}}$; i.e. the interval c contains the point 1.

- 5) Let $d = a \cdot b; e = a \cdot c; f = b + c; g = a \cdot f; h = d + e$. Then $\bar{f} = \bar{b} + \bar{c}; \varepsilon_f = \sqrt{\varepsilon_b^2 + \varepsilon_c^2}; \bar{g} = \bar{a} \cdot \bar{f} = \bar{a} \cdot \bar{b} + \bar{a} \cdot \bar{c}; \varepsilon_g = \sqrt{\bar{a}^2 \cdot \varepsilon_f^2 + \bar{f}^2 \cdot \varepsilon_a^2} = \sqrt{\bar{a}^2 \cdot \varepsilon_b^2 + \bar{a}^2 \cdot \varepsilon_c^2 + (\bar{b} + \bar{c})^2 \cdot \varepsilon_a^2}; \bar{d} = \bar{a} \cdot \bar{b}; \varepsilon_d = \sqrt{\bar{a}^2 \cdot \varepsilon_b^2 + \bar{b}^2 \cdot \varepsilon_a^2}; \bar{e} = \bar{b} \cdot \bar{c}; \varepsilon_e = \sqrt{\bar{a}^2 \cdot \varepsilon_c^2 + \bar{c}^2 \cdot \varepsilon_a^2}; \bar{h} = \bar{d} + \bar{e} = \bar{a} \cdot \bar{b} + \bar{a} \cdot \bar{c}; \varepsilon_h = \sqrt{\varepsilon_d^2 + \varepsilon_e^2} = \sqrt{\bar{a}^2 \cdot \varepsilon_b^2 + \bar{a}^2 \cdot \varepsilon_c^2 + \bar{b}^2 \cdot \varepsilon_a^2 + \bar{c}^2 \cdot \varepsilon_a^2};$

Therefore, $g \subseteq h$ provided $\bar{b} \cdot \bar{c} \leq 0$. The theorem is proved. Let x is interval, characterized by the center v and the widths of the interval σ .

Let an interval-valued function $U = f(x)$ be defined, the argument of which is an interval. The value of this function is the interval, which we denote as $U = (\bar{u}, \varepsilon_u)$, and which is determined by the formula:

$$\bar{u} = f(v), \quad \varepsilon_u = \left| \left(\frac{\partial f}{\partial x} \right) \right|_v \sigma. \tag{10}$$

Let x_1, x_2, \dots, x_n intervals, characterized by the centers v_1, v_2, \dots, v_n and with the corresponding width $\sigma_1, \sigma_2, \dots, \sigma_n$.

Let an interval-valued function $U = f(x_1, x_2, \dots, x_n)$ be defined, the arguments of which are intervals. The value of this function will be the interval, which we denote as $U = (\bar{u}, \varepsilon_u)$, and which is determined by the formula:

$$\begin{aligned} \bar{u} &= f(v_1, v_2, \dots, v_n) \tag{11} \\ \varepsilon_u &= \sqrt{\left(\frac{\partial f}{\partial x_1} \right)_1^2 \cdot \sigma_1^2 + \left(\frac{\partial f}{\partial x_2} \right)_2^2 \cdot \sigma_2^2 + \dots + \left(\frac{\partial f}{\partial x_n} \right)_n^2 \cdot \sigma_n^2} \end{aligned}$$

As it can be seen from the above formulae for differentiable functions, the computation of function values using the new interval mathematics (11) is more constructively due to finiteness of the arithmetic operations being performed. At the same time, computation of function values using "classical" interval mathematics requires solution of two optimization problems, for solving each of which in the general case it is necessary to do iterative calculations. Meanwhile there arise problems of the convergence of an iterative process and the choice of an initial point /9,10/.

4 Results of the Research

Let us consider the example from /5/: Let be necessary to calculate the distribution of voltages in the R - circuit, given in Figure 1. According to the node potential method we introduce conductivities $r_i = R_i$. The system of equations for determining the potentials u_1, u_2, u_3 has the form

$$\begin{pmatrix} r_1 + r_2 + r_3 & -r_3 & 0 \\ -r_3 & r_3 + r_4 + r_5 & -r_5 \\ 0 & -r_5 & r_5 + r_6 \end{pmatrix} \begin{pmatrix} u_1 \\ u_2 \\ u_3 \end{pmatrix} = \begin{pmatrix} vr_1 \\ 0 \\ 0 \end{pmatrix}$$

Let the circuit elements have the following nominal (rated) values: $r_1 = r_2 = r_4 = 0.1; r_3 = r_5 = 2.0; v = 6.3$. Technologically these values may be withstood not accurately but with some error. Moreover, they often depend on the temperature and other operation conditions. Suppose that the resulted face-values are withstood within $\pm 10\%$.

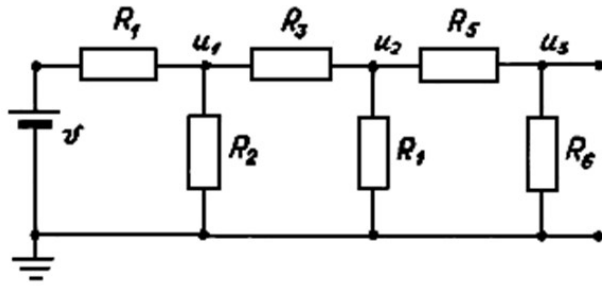


Figure 1: R – circuit.

Thus, really $r_1, r_2, r_4 \in [0.09, 0.11]$, $r_3, r_5 \in [1.8, 2.2]$, $v \in [5.67, 6.93]$.

At any combination of these data the matrix of the system is a nonsingular - matrix. Let us denote by A the interval matrix and by b the interval vector of the right parts of the system:

$$A = \begin{bmatrix} [1.98, 2.42] & [-2.2, -1.8] & [0, 0] \\ [-2.2, -1.8] & [3.69, 4.51] & [-2.2, -1.8] \\ [0, 0] & [-2.2, -1.8] & [1.89, 2.31], \end{bmatrix}$$

$b = \{[0.5103, 0.7623], [0, 0], [0, 0]\}$. According to the classical definition of arithmetic operations, the interval matrix is singular [11, 12]. Using the definitions of arithmetic operations introduced, we obtain the following values of potentials: $u_1 = [-0.108, 3.443] = (1.667, 1.775)$; $u_2 = [-0.422, 3.460] = (1.519, 1.941)$; $u_3 = [-0.737, 3.630] = (1.447, 2.183)$. Let us consider the example by Raikhman [4]. An interval matrix $S(\alpha)$ is given.

$$S(\alpha) = \begin{pmatrix} 1 & [0, \alpha] & [0, \alpha] \\ [0, \alpha] & 1 & [0, \alpha] \\ [0, \alpha] & [0, \alpha] & 1. \end{pmatrix}$$

For $(\sqrt{5} - 1)/2 \leq \alpha < 1$, using classical arithmetic operations, when calculating the determinant by Gauss’s method we will obtain that the interval matrix $S(\alpha)$ is singular.

When using the interval arithmetic operations introduced, which are realized in the form of a library of subroutines and functions, we will obtain the solutions of the system of linear interval algebraic equations

$$S(\alpha)x(\alpha) = \begin{pmatrix} [1, 1] \\ [1, 1] \\ [1, 1] \end{pmatrix}$$

for different values of α :

$$x(0.65) = \begin{pmatrix} [0.244, 0.968] \\ [0.132, 1.080] \\ [0.075, 1.137] \end{pmatrix} = \begin{pmatrix} [0.606, 0.362] \\ [0.606, 0.474] \\ [0.606, 0.531] \end{pmatrix},$$

$$x(0.70) = \begin{pmatrix} [0.187, 0.989] \\ [0.065, 1.111] \\ [-0.002, 1.178] \end{pmatrix} = \begin{pmatrix} [0.588, 0.401] \\ [0.588, 0.523] \\ [0.588, 0.590] \end{pmatrix},$$

$$x(0.80) = \begin{pmatrix} [0.060, 1.052] \\ [-0.077, 1.188] \\ [-0.166, 1.277] \end{pmatrix} = \begin{pmatrix} [0.556, 0.496] \\ [0.556, 0.533] \\ [0.556, 0.721] \end{pmatrix},$$

$$x(0.90) = \begin{pmatrix} [-0.095, 1.147] \\ [-0.236, 1.289] \\ [-0.350, 1.403] \end{pmatrix} = \begin{pmatrix} [0.526, 0.621] \\ [0.526, 0.762] \\ [0.526, 0.877] \end{pmatrix}.$$

5 Conclusion

In this paper we gave the definition of a new interval mathematics, investigated its properties. The new interval arithmetic including nonuniform distribution of values within an interval was introduced in the paper.

Using solutions of some classical problems, the efficiency of the introduced interval mathematics was shown.

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